LE34

BestFit User Guide

Dimensional Control at your fingertips

Part 1

LE34

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About BestFit

BestFit is an application developed by LE34 A/S to determine the best fitting geometrical shapes for use in controllers with Trimble Access software. It enables determining the parameters of well known geometric shapes in 3 dimensions as well as their basic intersections and projections. The results are stored as a job in the job folder, ready for setting out in the field.

The user guide id divided into Part 1 and Part 2. Part 1 is explaining some important issues and requirements and Part 2 is a step by step guide.

Shapes and Intersections

The basic geometric shapes are:

Shape	Minimum Required Datapoints	BestFit Definition within TrimbleAccess
Line	2 points	2 points and a unit vector
Plane	3 points, or 1 line and 1 point	Vector normal to the plane
Circle	3 points	Vector normal to the plane of the circle
Sphere	4 non-planar points	Center of the sphere
Cylinder	3 points on a plane + 1 point for direction of the cylinder on the surface	2 points and a unit vector

The possible intersections and the results generated by BestFit are:

Intersection	Required for a Result	Result
2 Lines	2 Non-parallel lines	1 point
2 Lines	2 Non-parallel, non-intersecting lines*	1 line
Line – Plane	1 Line non-parallel to 1 plane	1 point
2 Planes	2 Mutually non-parallel planes	1 Line as a vector along intersection
3 Planes	3 Mutually non-parallel planes	1 point
Line – Sphere	1 Line intersecting 1 sphere	2 points
Line – Cylinder	1 Line intersecting 1 cylinder	2 points

The perpendicular projections which can be determined by BestFit are:

Projection	Required for a Result	Result
Point onto a line	1 Line and 1 Point	1 point
Point onto a Plane	1 Line and 1 Point	1 Point

All Resultant Points are given using a three dimensional Cartesian Coordinate System, i.e. expressed as (x,y,z).

*Note: The BestFit intersection of 2 lines permits a result of either one or two points. The two point result defines a line mutually perpendicular to the 2 input lines. In other words, the resultant line is the minimum perpendicular offset between the two given lines.

Geometric Shapes - Resulting Lines

A BestFit Plane is expressed as a Normal. The two points defining this normal are separated by a distance of one unit, resulting in a unit vector perpendicular to the resultant plane. Resultant Vectors and Normals are the input for the calculations of intersections.

The cases in which no unit or normal vector is calculated are:

- A single line result as the result is the line itself as given by its start and end points.
- A sphere, as the radius may be considered either directionless or with an infinite number of directions.

BestFit - The Calculation

The BestFit calculation is based on a robust Least Squares determination of the unknown parameters of the shapes. The calculation report includes values for the unknowns, including values for the estimated accuracy (via RMS) as given by the residual values with respect to the best-fitting shape.

The RMS (Root Mean Square) Deviation values correspond to the standard deviation between the points shot in the field and the BestFit geometry.

In mathematics, the root mean square (RMS), also known as the quadratic mean, is a statistical measure of the magnitude of a varying quantity may be determined using a formula such as:

$$x_{\mathrm{rms}} = \sqrt{\frac{1}{n}\left(x_1^2 + x_2^2 + \cdots + x_n^2\right)}.$$

Glossary

Adapted from Wikipedia

Best Fit
The best fit in the least-squares sense minimizes the sum of squared residuals.
See also **BestFit – The Calculation** above.

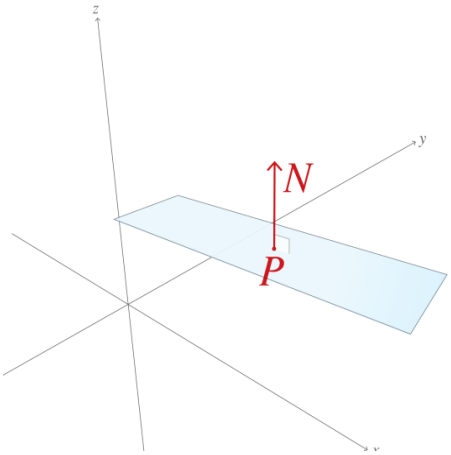
$$S = \sum_{i=1}^{n} r_i^2$$

Least Squares

The method of least squares is a standard approach to the approximate solution of over determined systems, i.e., sets of equations in which there are more equations than unknowns. "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in the results of every single equation.

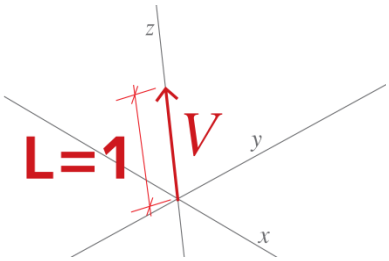
The least squares method finds its optimum when the sum, *S*, of squared residuals is minimum.

Normal
In three-dimensional space, a surface may also be defined by its normal, ***N***.
The normal at a point ***Π*** on a plane is a vector perpendicular to the plane with an origin of ***P***.
Normal is also used as an adjective: a line normal to a plane etc.



Residual
Residual is the difference between an observed value and the fitted value provided by a model or shape.
The sum of the residuals as given by the RMS (see above) may be used as a means to judge how well a best-fit result conforms to the given data set.

Unit vector
A unit vector ***V*** is a line in Cartesian space with both magnitude and direction, and whose magnitude (or length) is 1 unit.
A unit vector differs from an ordinary vector as it shares its starting point with the origin of the given coordinate system (0,0,0).



Nomenclature

Calculation results, i.e. resultant points and lines are stored in the job. The names and feature codes are automatically generated by the application. The naming conventions are given in the table below:

Geometric Shape	Resultant Point(s)	Description of Result	Resultant Line	Feature Code
Circle	Cir1_A*	Circle Centre, unit vector start point	Line1	Cir1_A-Cir1_B
	Cir1_B	Circle, unit vector end point		
Plane	Pla1_A	Center of Gravity, unit vector start point	Line1	Pla1_A-Pla_B
	Pla1_B	Plane, unit vector end point		
Line	Lin1_A	Line, start point	Line1	Lin1_A-LinB
	Lin1_B	Line end point		
Sphere	Sph1	Sphere, Centre point	N/A	Sph1_Centre
Cylinder	Cyl1_A	Cylinder, Centerline start point	Line1	Cyl1_A-Cyl1_B
	Cyl1_B	Cylinder, Centerline end point		
Intersection of 2 lines	X1_A	Int. 2 lines, line1-Line2, point on Line 1	N/A	Lin1_Lin2
	X1_B	Int. 2 lines, line1-Line2, point on Line 2		
Intersection of Plane - line	X1	Int. Plane-line, point of intersection	N/A	Lin1_Lin2
Intersection of 2 planes	X2_A	Int. 2 planes, unit vector start point	Line1	Lin1_Lin2
	X2_B	Int. 2 planes, unit vector end point		
Intersection of 3 planes	X3	Int. 3 planes, point of intersection	N/A	Lin1_Lin2_Lin3
Intersection of a line and a sphere	X4_A X4_B	2 Points on the sphere	N/A	Lin1_Sph1
Intersection of a line and a cylinder	X5_A X5_B	2 Points on the cylinder	N/A	Lin1_Cyl1
Projection of a point onto a line	P1	Point on the line	N/A	Lin1_Prj
Projection of a point onto a plane	P2	Point on plane	N/A	Pla1_Prj

* Please note that the cardinal designation 1 applies to the first Circle (or shape) calculated. The following shape(s) will follow in numerical order. Numbers assigned in the above table are for illustration only.

This is valid for all shape names, and has no relation to the shapes or their possible intersection types.

The BestFit Shapes

The results of the calculations are included in a report stored in the job folder.

Following are the output specifications for the following BestFit Shapes :

- **BestFit Line**
- **BestFit Plane**
- **BestFit Circle**
- **BestFit Sphere**
- **BestFit Cylinder**

BestFit Line

A 3D line best fitting the given points, defined by points ***a*** and ***b*** which are determined by the projection of the two outermost points onto the BestFit line.

Stored in the Report:

- The BestFit line start and end points ***a*** and ***b***
- Length of the line ***L***
- A unit vector ***V*** giving the orientation of the line
- The RMS factor, giving the perpendicular residual from the points to the line

Stored in the job for setting out:

- The line ***a-b***
- The points ***a*** and ***b***

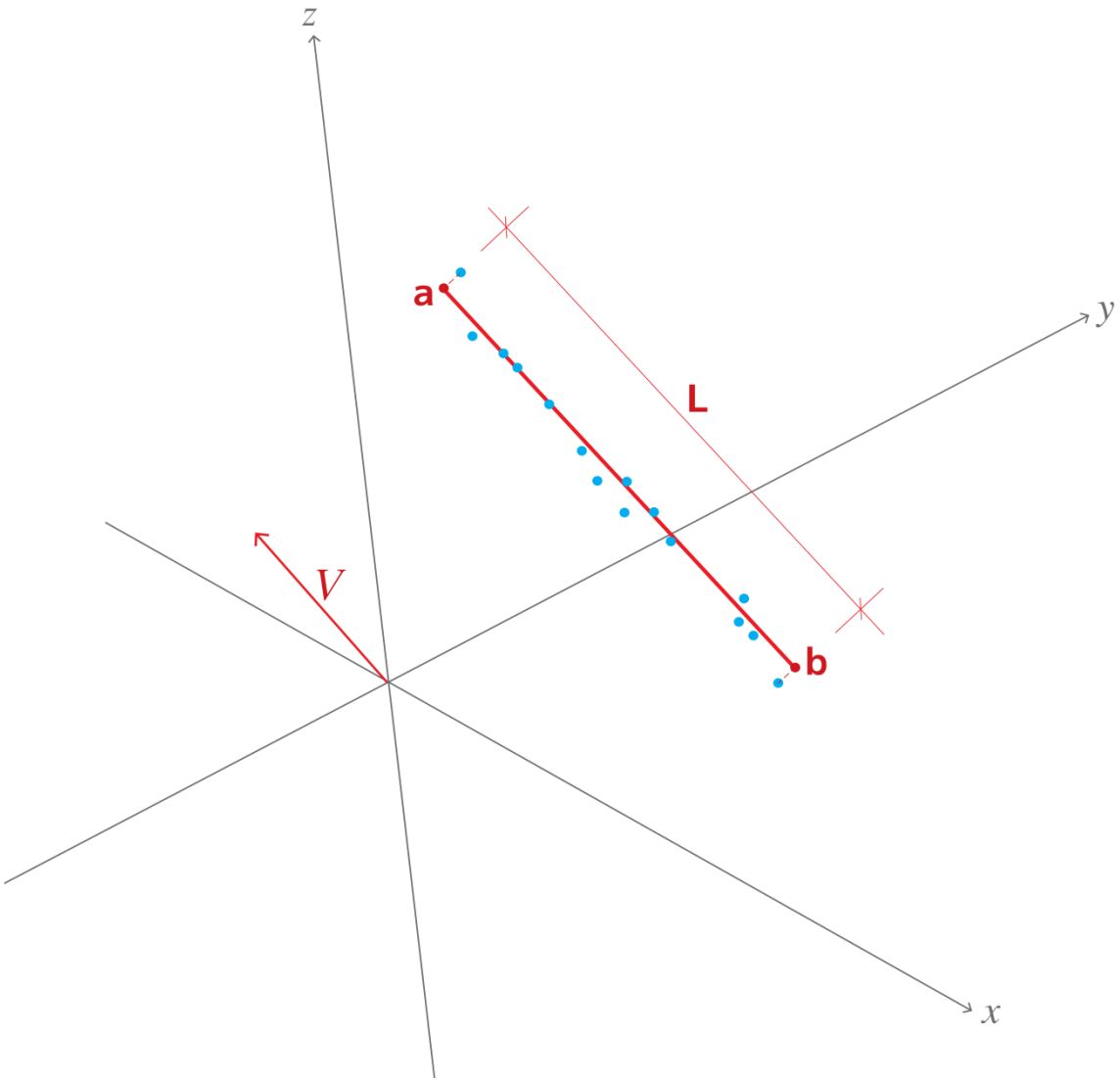


Fig 1: BestFit Line

BestFit Plane

The plane which best fits the given points. The plane is defined by a normal ***N*** with an origin at the center of gravity of the given points.

Stored in the Report:

- The centre of gravity of the given points at point ***a***
- A normal ***N*** (defined by the two points ***a*** and ***b***)
- A unit vector ***V*** identical in orientation to the normal ***N***
- A list of the perpendicular residuals from the points to the plane

Stored in the job for setting out:

- The line ***a-b***
- The points ***a*** and ***b***

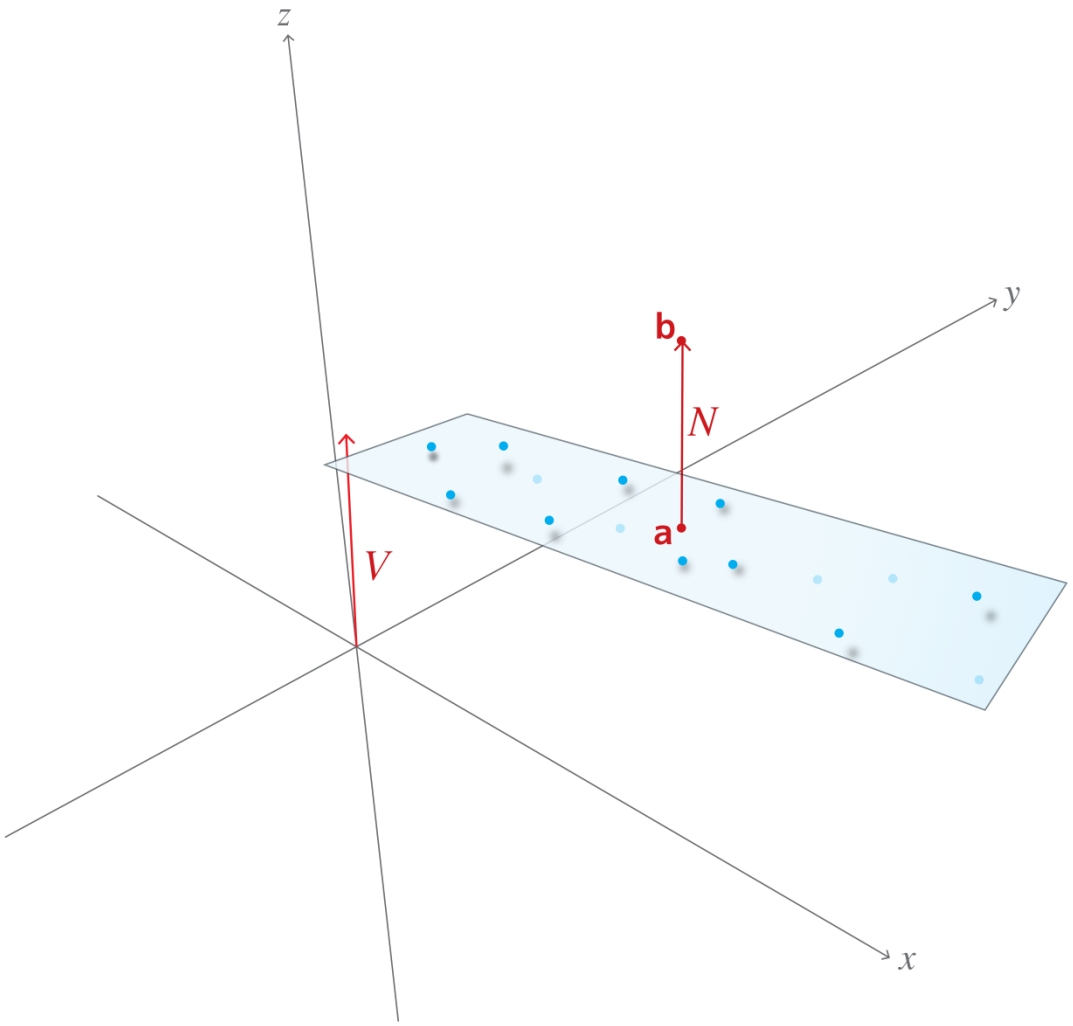


Fig 2: BestFit Plane

BestFit Circle

The circle which best fits the given points.

The circle is defined by a radius ***R*** and has an orientation given by the normal ***N*** with origin at the center of the resultant circle.

Stored in the Report:

- The centre ***a,*** and radius ***R*** of the circle
- The normal ***N*** for the circle (defined by the two points ***a*** and ***b***, where point ***b*** is one unit from ***a***)
- A list of the perpendicular residuals from the points to the circle
- A list of the radial residuals of the points

Stored for setting out:

- The line ***a-b***
- The points ***a*** and ***b***

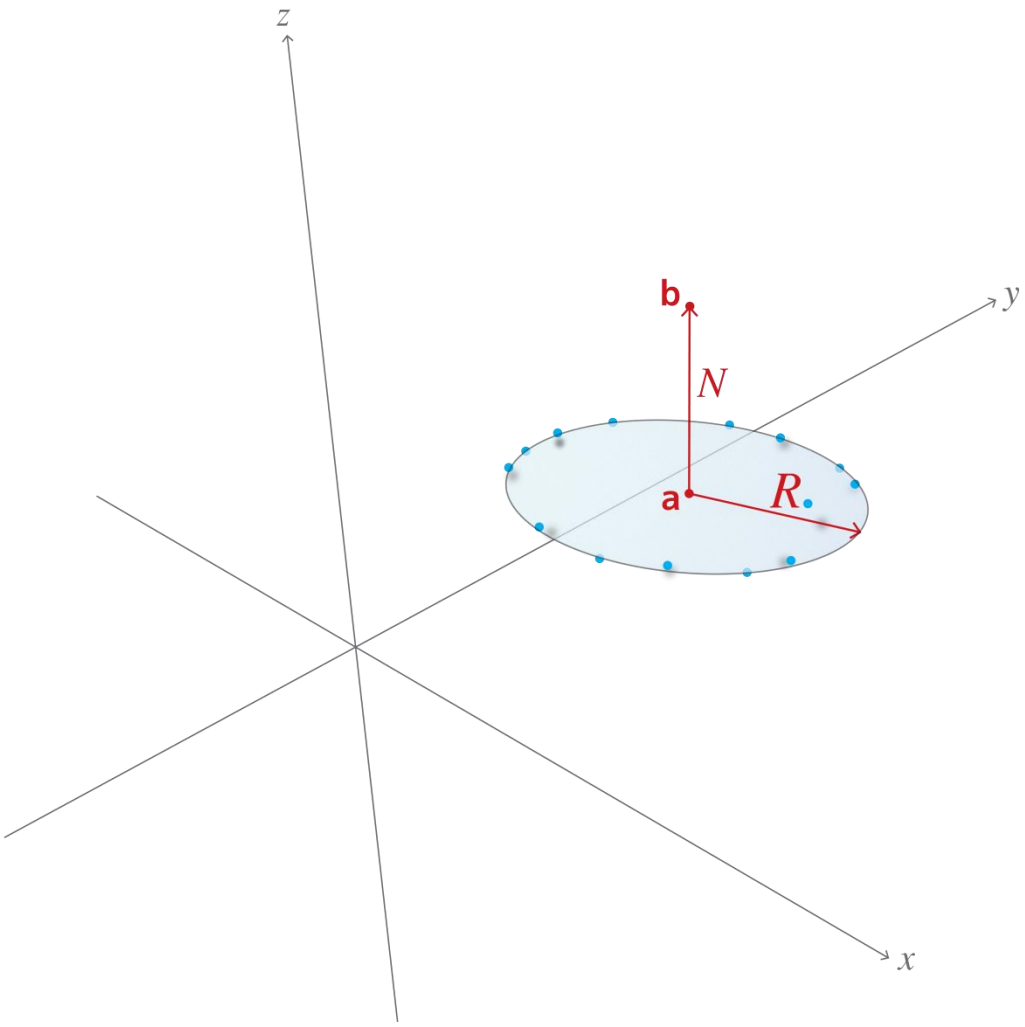


Fig 3: BestFit Circle

BestFit Sphere

The sphere which best fits the given points. The sphere is defined by a radius ***R*** at the center ***a*** of the resultant sphere.

Stored in the Report:

- The centre ***a*** and radius ***R*** of the sphere
- A list of the radial residuals of the points

Stored for setting out:

- The point ***a***

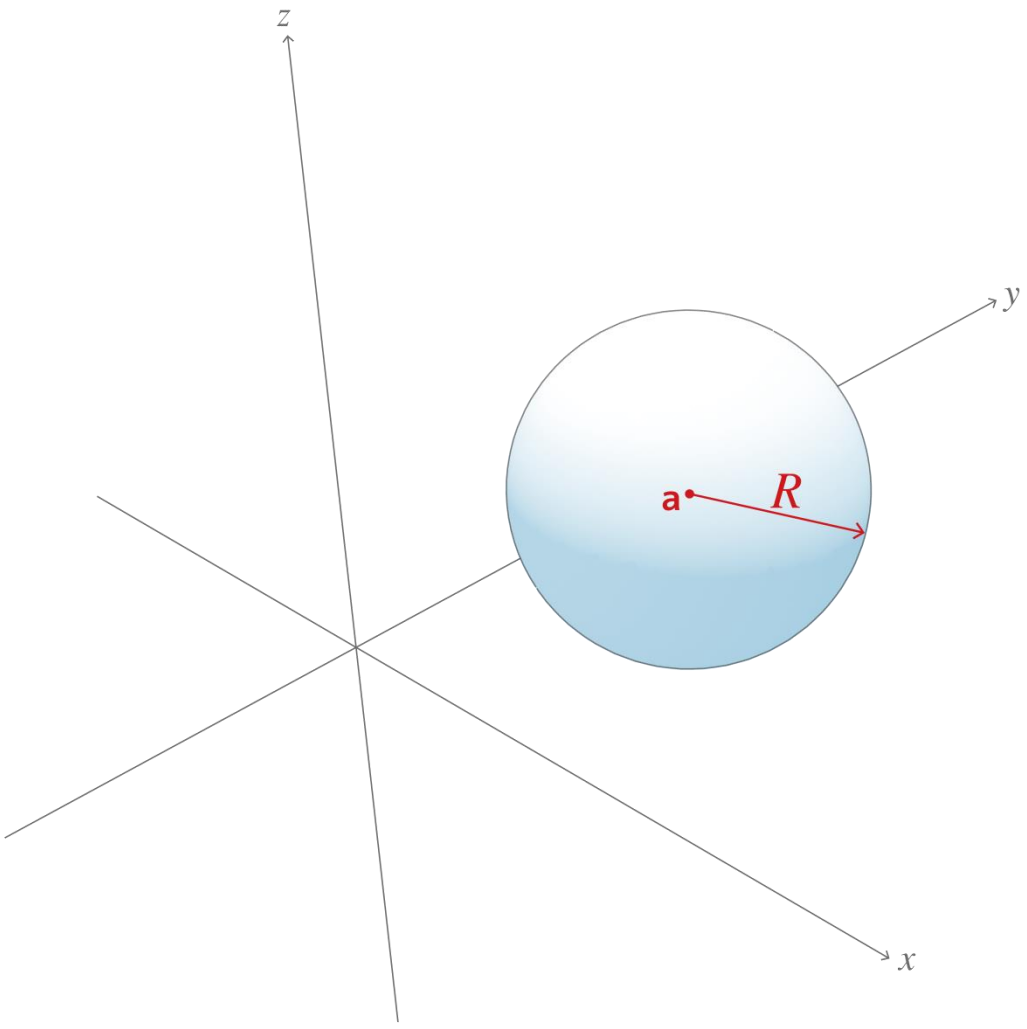


Fig 4: BestFit Sphere

BestFit Cylinder

The cylinder which best fits the given points. A minimum of 7 points on the cylinder wall are required. The points shall be surveyed along the edge of the cylinder (one side at the time) and some points along the middle of the cylinder. A radius R and a vector C along the axis of the resultant cylinder define the cylinder.

Stored in the Report:

- The radius ***R*** of the cylinder
- Axis and orientation of the cylinder defined by the two outermost points ***a*** and ***b***, as determined by projection onto the axis of the cylinder,
- A list of the radial residuals between the given points and the cylinder

Stored for setting out:

- The line ***a — b***
- The points ***a*** and ***b***

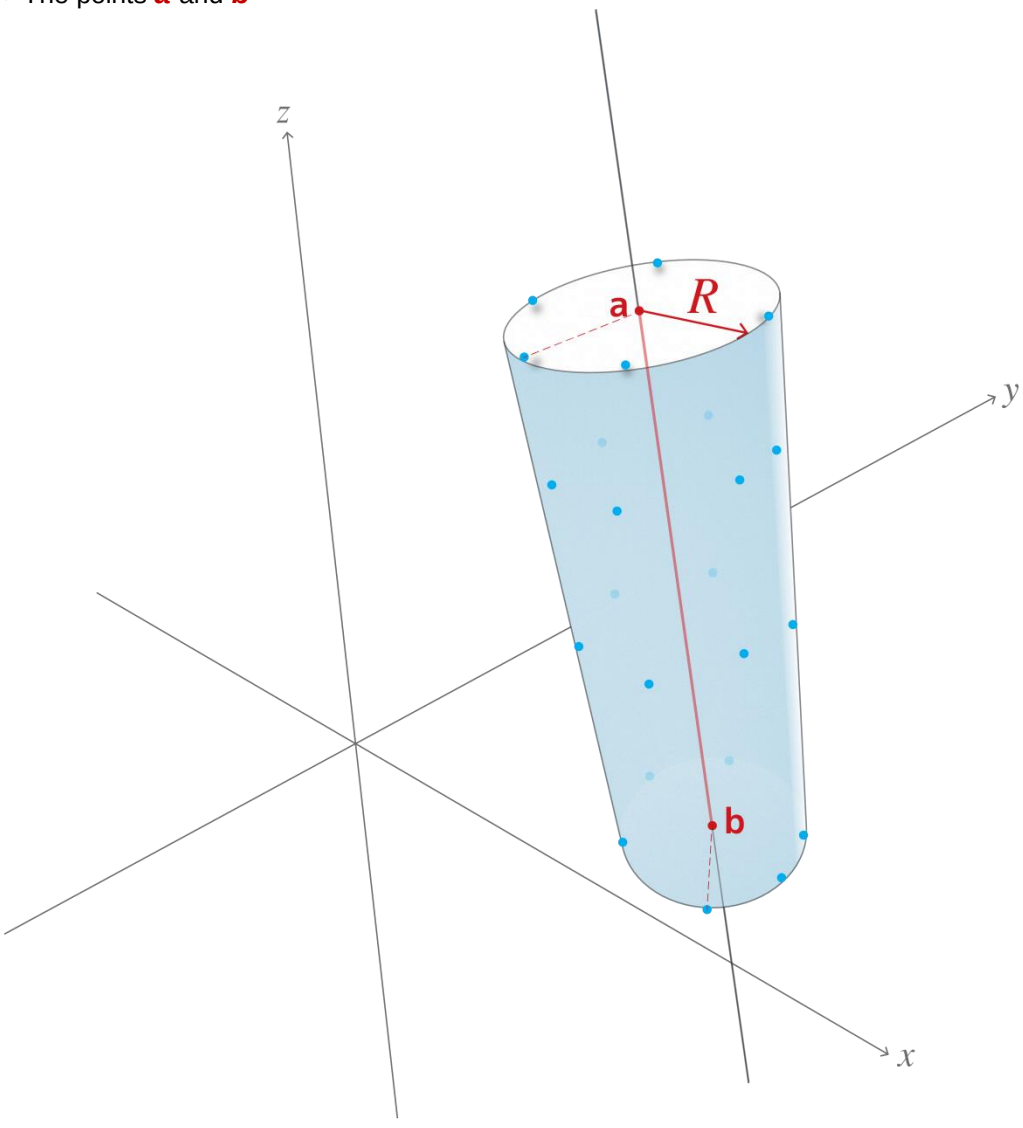


Fig 5: BestFit Cylinder

The BestFit Intersections and projections

The results of the calculations for BestFit Intersections are included in a report stored in the job folder.

Following are the output specifications for the following BestFit Intersections:

- **Intersection of 2 Lines**
- **Intersection of a Line and a Plane**
- **Intersection of 2 Planes**
- **Intersection of 3 Planes**
- **Intersection of a Line and a Sphere**
- **Intersection of a Line and a Cylinder**

Following are the output specifications for the following BestFit Intersections:

- **Projection of a point onto a line**
- **Projection of a point onto a Plane**

Intersection of 2 Lines

Stored in the Report:

- A single point ***a*** , collinear with both lines in the case of geometric intersection, ***or***
- Two points ***a*** and ***b*** on defining the shortest mutually perpendicular line ***L*** between the given lines
- The length of ***L***

Stored for setting out:

- a single point ***a***, ***or***
- 2 points, ***a*** and ***b***

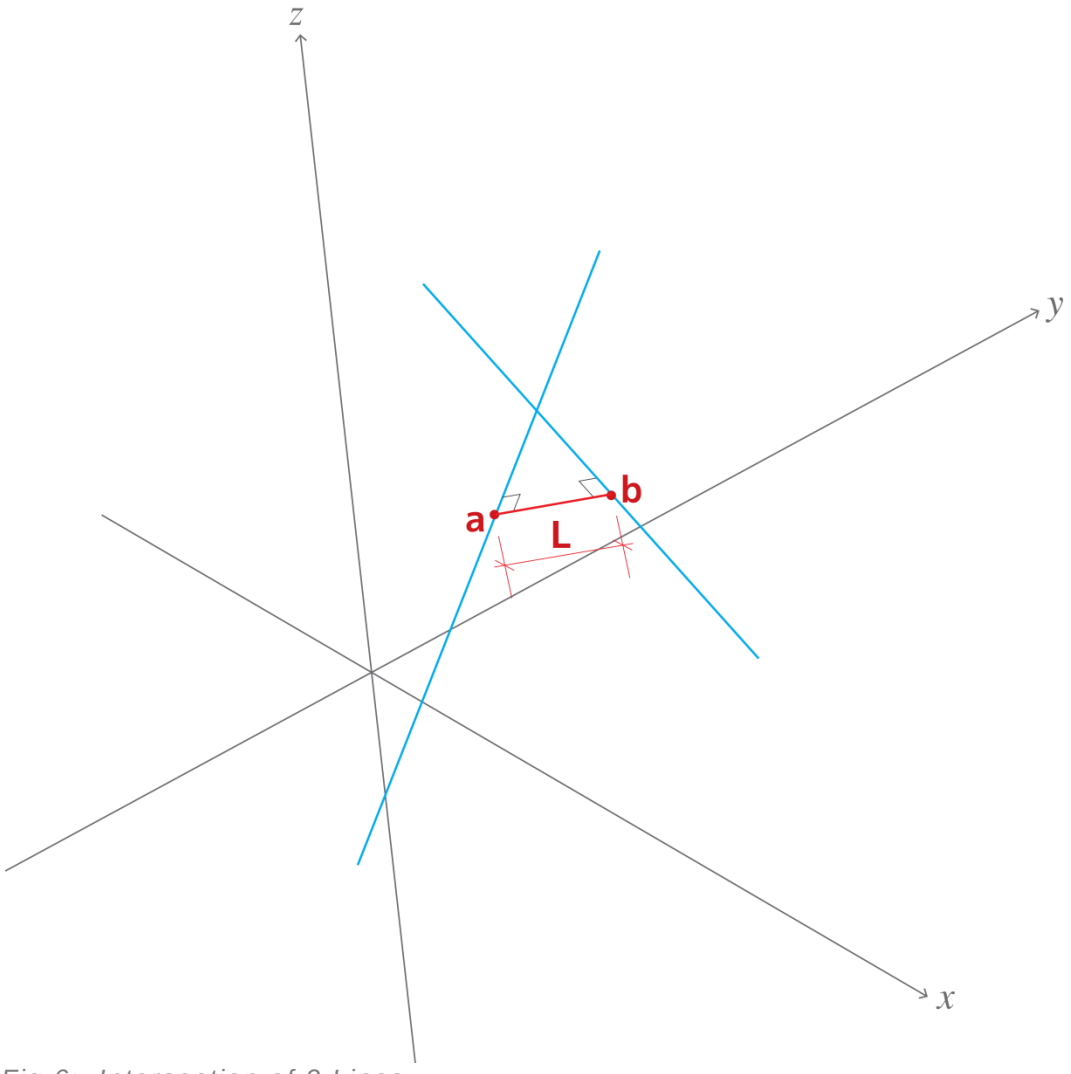


Fig 6: Intersection of 2 Lines

Intersection of a Line and a Plane

Stored in the Report:

- The point ***a*** at the intersection of the line and plane

Stored for setting out:

- The point ***a***

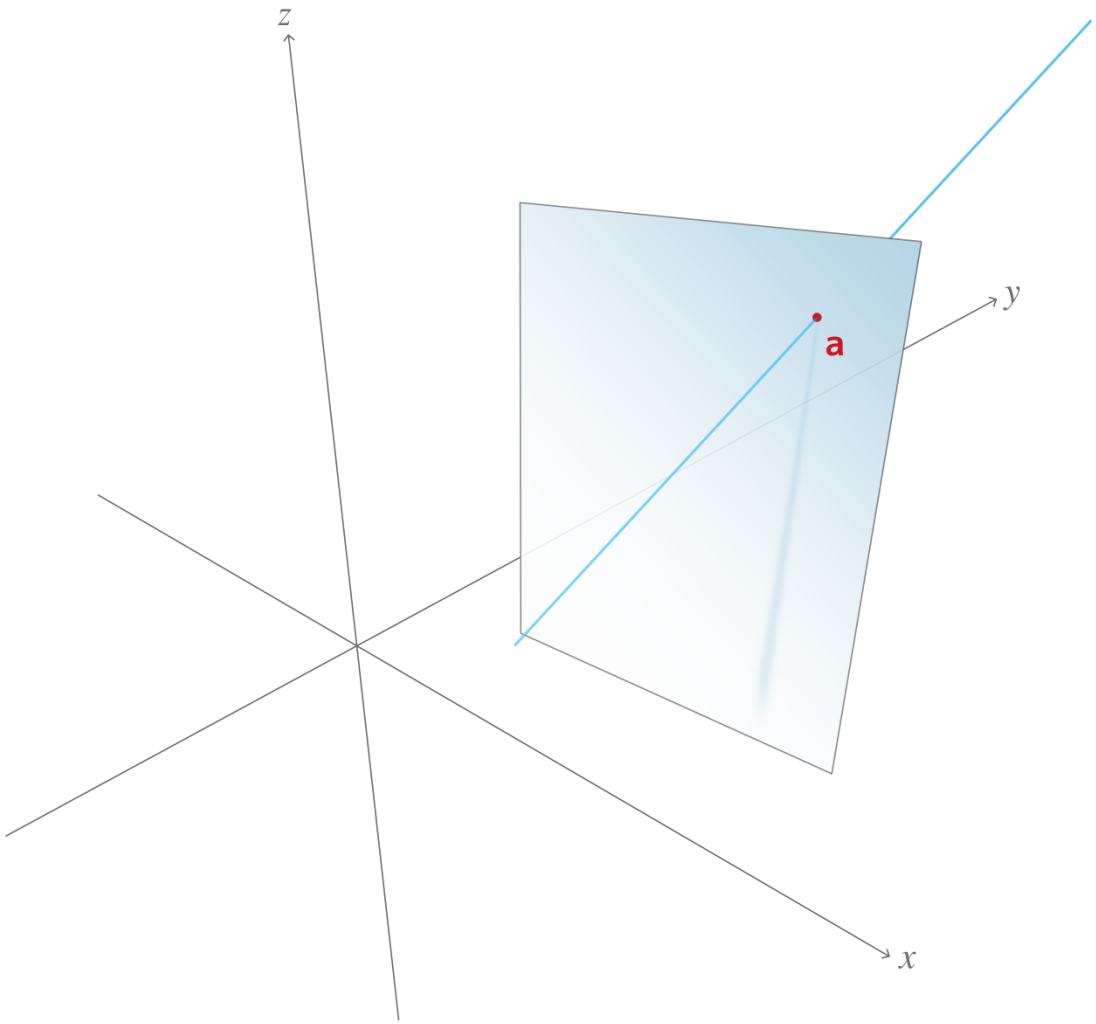


Fig 7: Intersection of line and plane

Intersection of 2 planes

Stored in the Report:

- A vector with length of one along the intersection of the two planes.
- The two points ***a*** and ***b***

Stored for setting out:

- The line ***a — b***
- The points ***a*** and ***b***

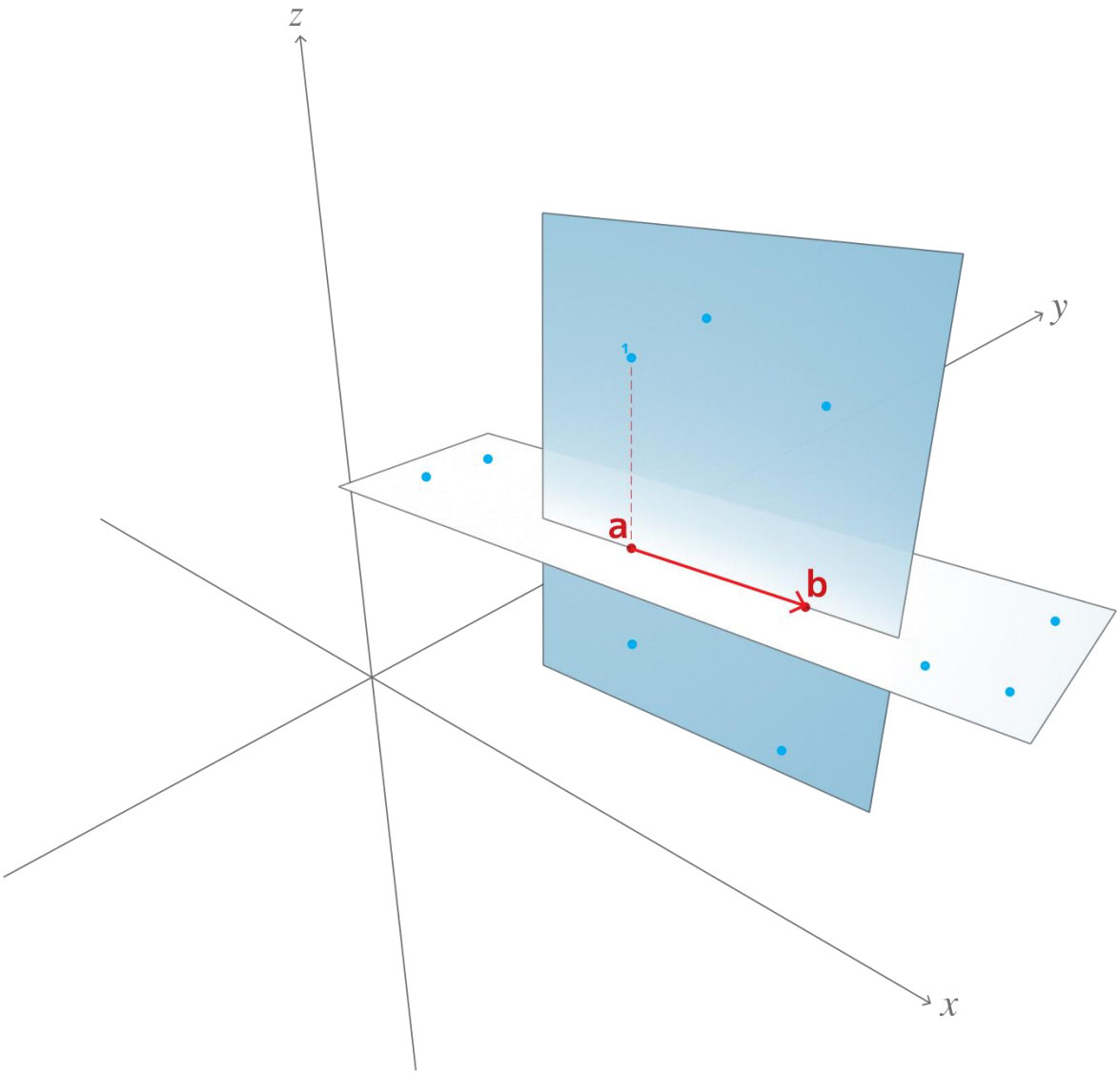


Fig 8: Intersection of 2 planes

Intersection of 3 planes

Stored in the Report:

- The point ***a*** at the intersection of the three planes. ***

Stored for setting out:

- The point ***a***

***** Note: There are several situations where intersection of three planes does not result in a single point. BestFit will only generate a valid result when the intersection of three planes is a single point.

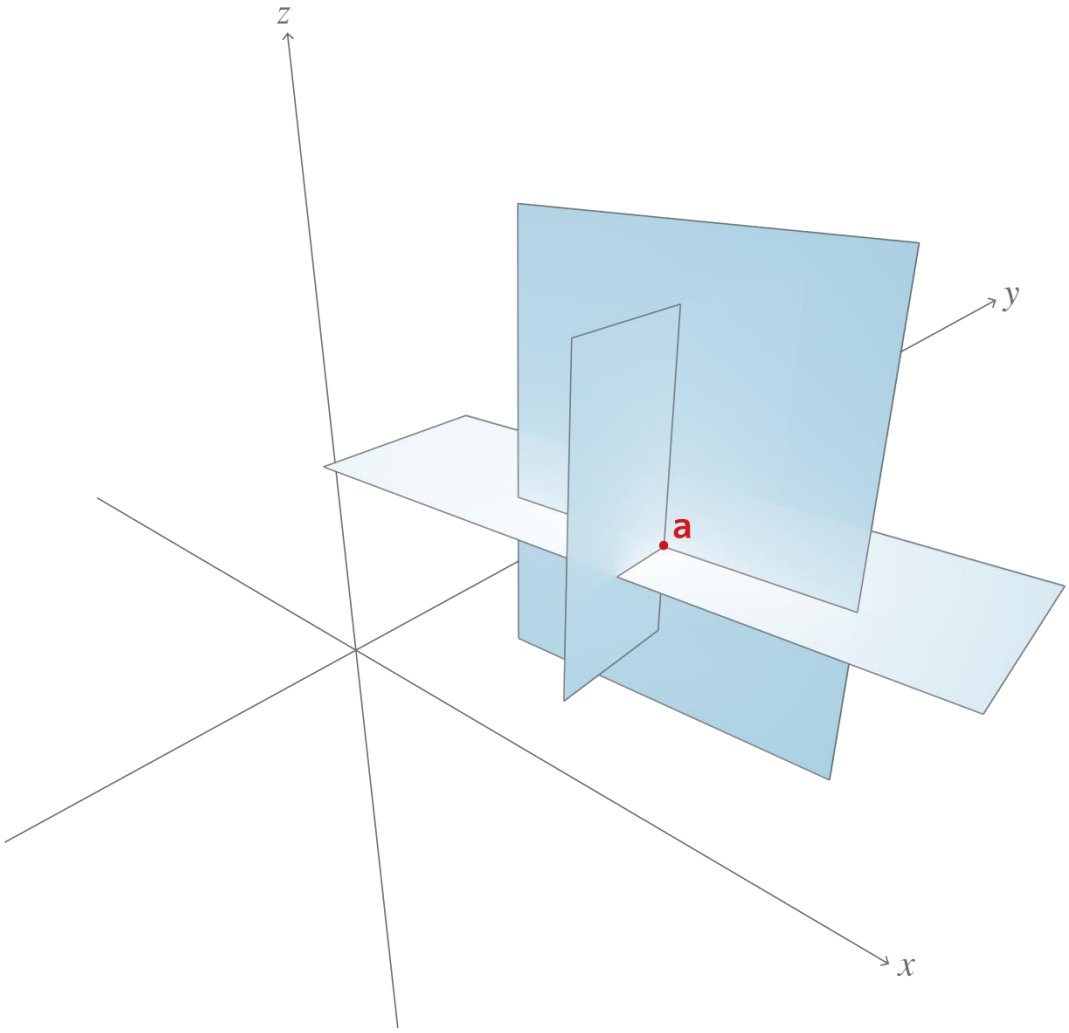


Fig 9: Intersection of 3 planes

Intersection of a Line and a Sphere

Stored in the Report:

- Two points **a** and **b** at the intersection of the line with the sphere

Stored for setting out:

- The two points **a** and **b** at the intersections of the line with the sphere.

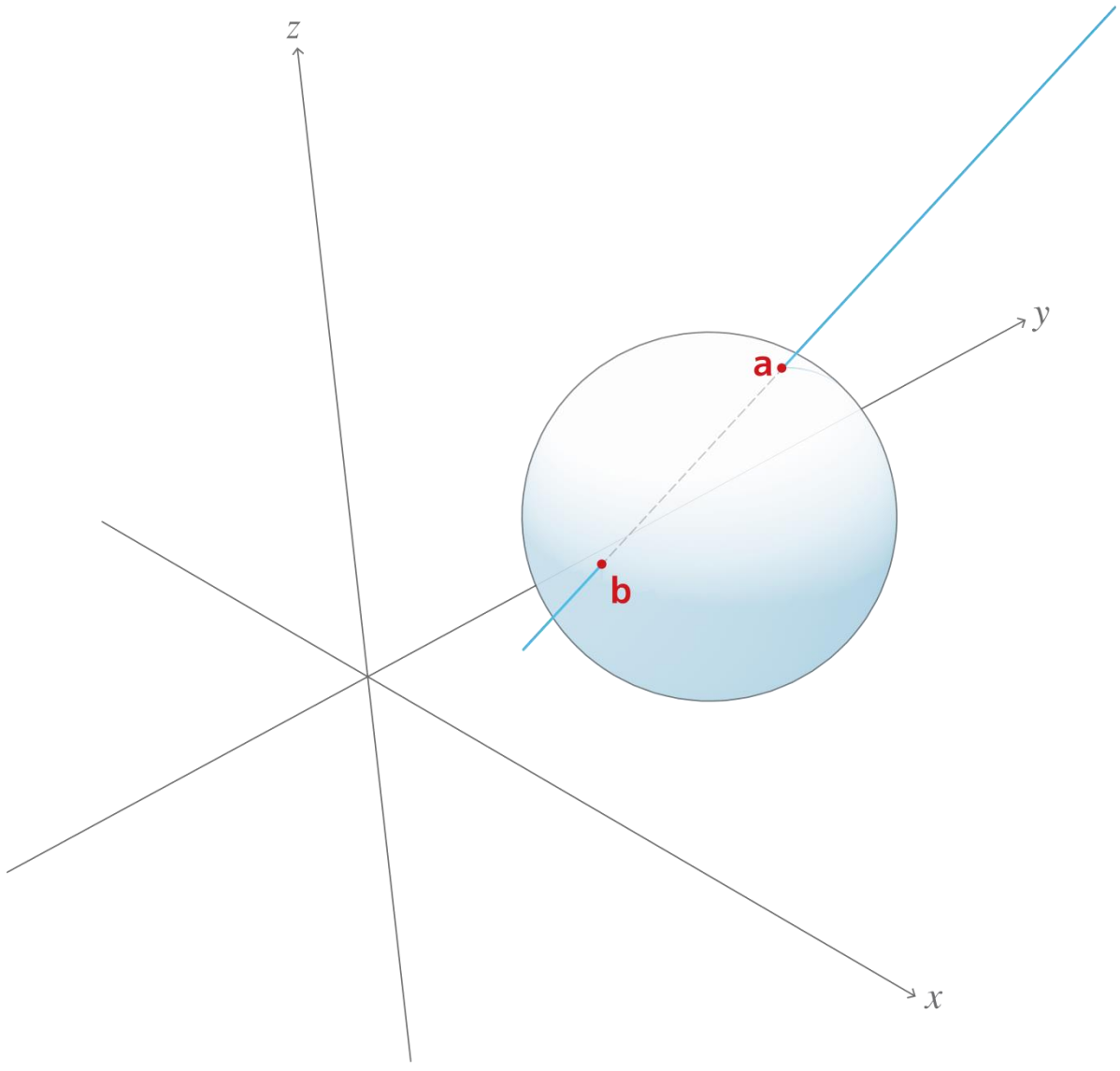


Fig 10: Intersection of line and sphere

Intersection of a Line and a Cylinder

Stored in the Report:

- Two points **a** and **b** at the intersection of the line and the cylinder wall

Stored for setting out:

- The two points **a** and **b** found at the intersections of the line and cylinder.

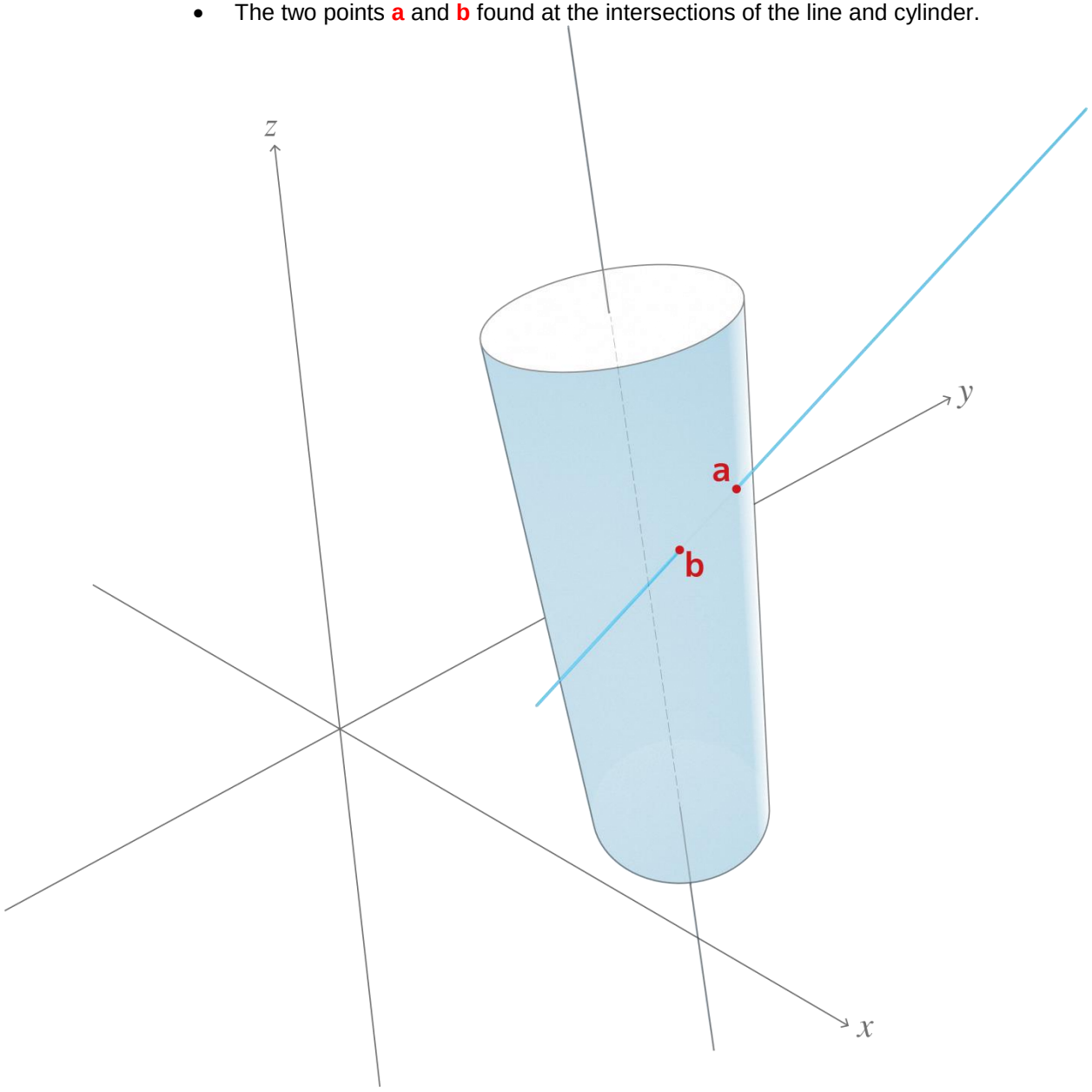


Fig 11: Intersection of line and a cylinder

Projection of a point onto a Line

Stored in the Report:

- A single point ***a*** on the line

Stored for setting out:

- Point ***a***

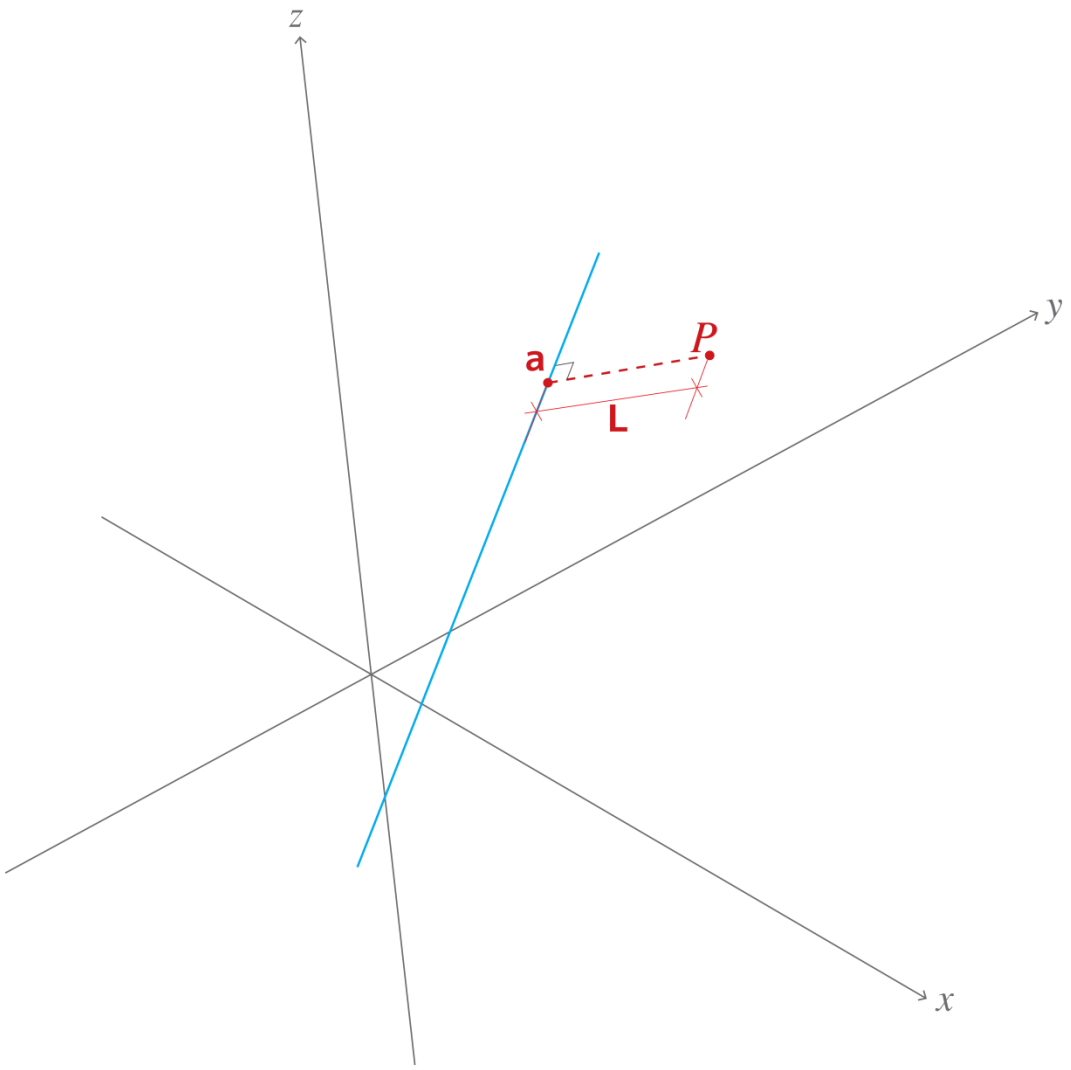


Fig 12: Projection of a point onto a line

Projection of a point onto a Plane

Stored in the Report:

- A single point ***a*** on the plane

Stored for setting out:

- Point ***a***

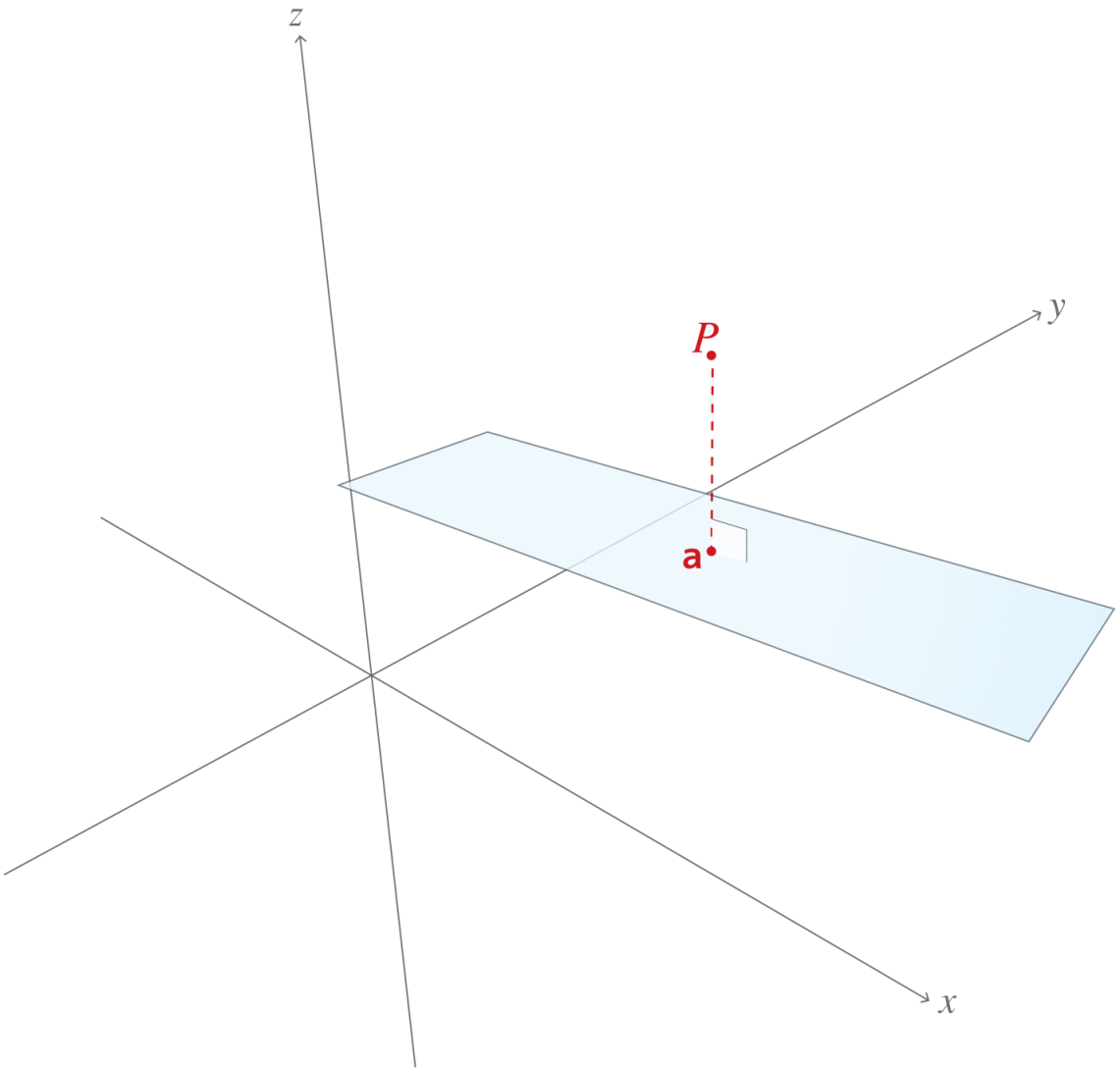


Fig 12: Projection of a point onto a plane